

# PROCEEDINGS OF SPIE

[SPIDigitalLibrary.org/conference-proceedings-of-spie](https://SPIDigitalLibrary.org/conference-proceedings-of-spie)

## Performance evaluation for imaging with a vortex half-wave retarder

Zhang, Yongqi, Wang, Wei

Yongqi Zhang, Wei Wang, "Performance evaluation for imaging with a vortex half-wave retarder," Proc. SPIE 11548, Optical Design and Testing X, 1154818 (10 October 2020); doi: 10.1117/12.2573321

**SPIE.**

Event: SPIE/COS Photonics Asia, 2020, Online Only

# Performance Evaluation for Imaging with a Vortex Half-Wave Retarder

Yongqi Zhang<sup>a</sup> and Wei Wang<sup>a,b</sup>

<sup>a</sup> School of Optoelectronic Engineering, Xi'an Technological University, Xi'an, Shaanxi, China

<sup>b</sup> School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, UK  
[w.wang@hw.ac.uk](mailto:w.wang@hw.ac.uk)

## ABSTRACT

A vortex half-wave retarder (VHR) is a new type of polarizing element with a constant retardance across its clear aperture but its fast axis rotating continuously over the area of the optic. In polarization optics, the VHR-based polarization control method is very efficient to control the radial and azimuthal polarization states of light with a simple system configuration, ease of use, and high energy utilization efficiency. In optical manipulation, VHR can generate nondiffracting Bessel beams with an enlarged trapping region of optical tweezers. In the field of optical imaging, an imaging system with a vortex half-wave retarder has been reported to improve the resolution. Due to its many unique functions with novelty, vortex half-wave retarder has received a lot of interests in optical micro-operation, optical imaging, optical communication, optoelectronics, quantum information and remote sensing. In this paper, we study the performance evaluation for imaging with a 0-order vortex half-wave retarder by using a method referred to as Optical Transfer Matrix. After introduction of the Jones matrix for the vortex half-wave retarders as a general pupil matrix, we present the optical transfer matrix as the frequency transfer characteristics for the imaging system. As compared with a polarization imaging with a half-wave plate, the imaging system with a vortex half-wave retarder has a typical effect of apodizing by increasing a contrast for the high-frequency end.

**Keywords:** Optical Transfer Matrix, zero-order vortex half-wave retarder, super-resolution

## 1. INTRODUCTION

The use of vortex half-wave retarder (VHR) has attracted much attention for its applications in optical micro-operation, optical imaging, optical communication, optoelectronics, quantum information and so on, due to its unique structure with a constant retardance across its clear aperture but its fast axis rotating continuously over the area of the optic<sup>[5]</sup>. In polarization optics, the VHR-based polarization control method is very efficient to control the radial and azimuthal polarization states of light with a simple system configuration, ease of use, and high energy utilization efficiency<sup>[7]</sup>. In the case of radially or azimuthally polarized beams, they have been shown to improve performance in confocal microscopy<sup>[1]</sup> and lithography systems<sup>[2]</sup>. In optical manipulation, VHR can generate nondiffracting Bessel beams<sup>[4]</sup> with an enlarged trapping region of optical tweezers, and convert standard TEM<sub>00</sub> Gaussian beams into so-called "donut hole" Laguerre-Gaussian modes<sup>[8]</sup>. The Laguerre-Gaussian mode beam carries different orbital angular momentum, which can be coded to apply to optical communication and optical information. Secondly, intensity distribution and spiral phase structure can also be applied to the generation of orbital angular momentum entangled states, optical trap, optical wrench, atomic capture, particle manipulation and so on. In the field of optical imaging, an imaging system with a vortex half-wave retarder has been reported to improve the resolution. It is of great significance for the researches of optical imaging. Optical Transfer Matrix (OTM), as a method to analyze the performance of optical polarization imaging systems, enjoys some distinct advantages, such as more objective, more reliable, and it can be applied to both small and large aberration optical systems<sup>[9]</sup>. The OTM describes the frequency transfer characteristic of the system for each Stokes parameter propagating from the object plane to the image plane. The change of resolution can be obtained, and the performance can be evaluated by analyzing the image of OTM.

In this paper, we study the performance evaluation for imaging with a 0-order vortex half-wave retarder by using a method referred to as Optical Transfer Matrix. This paper is structured as follows. The model of an imaging system with a vortex half-wave retarder is set up in Section 2. In Section 3, after introduction of the Jones matrix for the vortex half-wave retarders as a general pupil matrix, we present the optical transfer matrix as the frequency transfer characteristics

for the imaging system. Section 4 gives the image of optical transfer matrix. In Section 5, as compared with a polarization imaging with a half-wave plate, the imaging system with a vortex half-wave retarder has a typical effect of apodizing by increasing a contrast for the high-frequency end. Conclusions are presented in Section 6.

## 2. MODLE OF POLARIZATION IMAGING SYSTEM WITH A VHR

A polarization imaging system is functional combination of an imager and a polarimeter, which is used to map the state of polarization(SOP) of the target. The SOP is commonly represented with Stokes vector for this system. The polarization imaging system with a VHR typically includes an imaging len and a 0-order vortex half-wave retarder, as shown in Figure 1. In addition, the system also includes imaging targets and image detectors. The distribution of Stokes vector is computed from the intensity images with polarization information.

Throughout this paper, we use a polarization imaging system with a 0-order vortex half-wave retarder, a rotating polarizer and an imaging len as a sample. The imaging performance of the vortex half-wave retarder is obtained by studying the optical transfer matrix of this polarization imaging system.

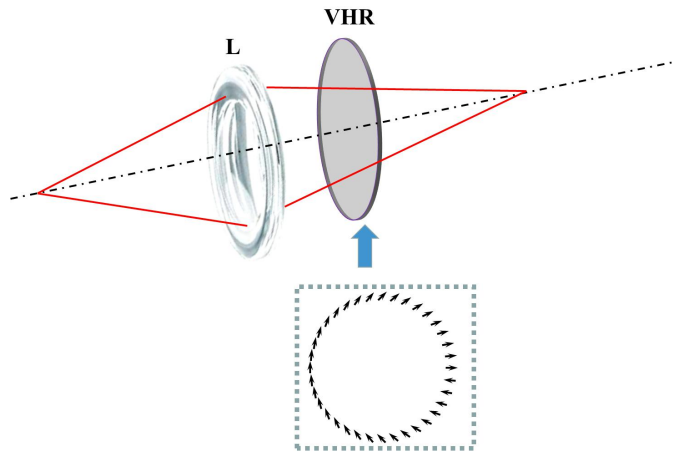


Figure 1. Structural diagram of an optical polarization imaging system with a 0-order vortex half-wave retarder

## 3. OPTICAL TRANSFER MATRIX OF A ZERO-ORDER VHR

Optical Transfer Matrix (OTM) is a  $4 \times 4$  matrix, and  $M_{mn} (m = 0 \sim 3, n = 0 \sim 3)$  are the elements of the OTM, Each element of the matrix is a Optical Transfer Function(OTF).

The Jones matrix for a half-wave plate is

$$J_{(x,y)} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} \quad (1)$$

When this half-wave plate is a 0-order vortex half-wave retarder, the elements of the Jones matrix are as follow<sup>[6]</sup>.

$$J_{xx} = \begin{cases} \cos(\text{atan2}(y,x)), & \sqrt{x^2 + y^2} \leq 1/2 \\ 0, & \sqrt{x^2 + y^2} > 1/2 \end{cases} \quad (2)$$

$$J_{xy} = \begin{cases} \sin(\text{atan2}(y,x)), & \sqrt{x^2 + y^2} \leq 1/2 \\ 0, & \sqrt{x^2 + y^2} > 1/2 \end{cases} \quad (3)$$

$$J_{yx} = \begin{cases} \sin(\text{atan2}(y, x)), & \sqrt{x^2 + y^2} \leq 1/2 \\ 0 & , \sqrt{x^2 + y^2} > 1/2 \end{cases} \quad (4)$$

$$J_{yy} = \begin{cases} -\cos(\text{atan2}(y, x)), & \sqrt{x^2 + y^2} \leq 1/2 \\ 0 & , \sqrt{x^2 + y^2} > 1/2 \end{cases} \quad (5)$$

The key to evaluating the polarization imaging system using the method of optical transfer matrix is to get the expression of each element in the optical transfer matrix. The expression of the optical transfer matrix of a polarization imaging system can be obtained by correlation operations from the Jones vector in its Jones matrix<sup>[10]</sup>. In next section, the image of OTM can be obtained by the expressions of OTM.

#### 4. THE IMAGE OF OPTICAL TRANSFER MATRIX

The expression of each element in the optical transfer matrix is obtained, so we can draw an image of each element of the optical transfer matrix in matlab software, as shown in Figure 2. the x axis represents the normalized spatial frequency in the x direction, which is represented by the formula  $f_x/2f_0$ .  $f_x$  represents a certain spatial frequency of this system in the x direction,  $2f_0$  represents the cutoff frequency of this system in the case of incoherent imaging. Similarly, the y-axis represents the normalized spatial frequency in the y direction. Because  $f_x/2f_0, f_y/2f_0$  represents the normalized spatial frequency. We define the value range of x and y as (-1, 1). The z axis represents the value of each element of the optical transfer matrix, and represents the change of the system's transfer capability with the change of spatial frequency. Since the entire system including the x-axis and y-axis are normalized, the maximum value of the optical transfer matrix will not exceed 1. We define the value range of the z-axis as (-1, 1). Another reason for normalization is to facilitate comparison with the optical transfer matrix of other systems.

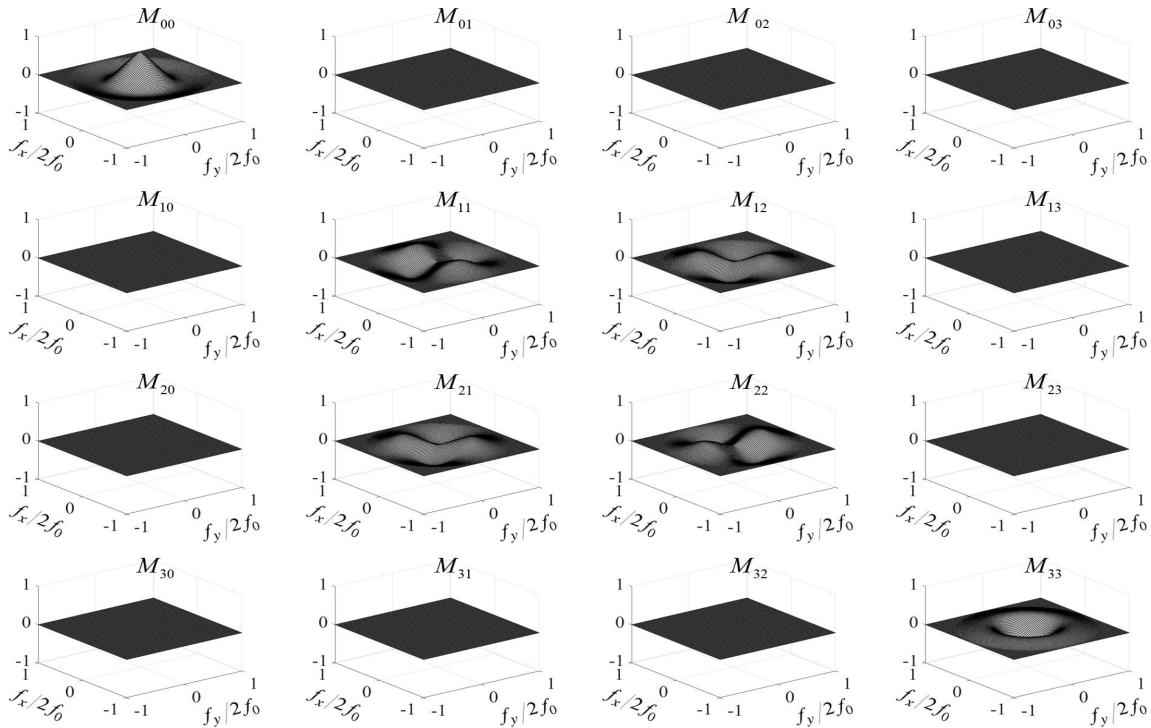


Figure 2. Transfer matrix image of polarization imaging system with VHR

As shown in Figure 2 above, The value of  $M_{01}, M_{02}, M_{03}, M_{10}, M_{13}, M_{20}, M_{23}, M_{30}, M_{31}, M_{32}$  is 0 or infinitely close to 0, which means that the zero-order vortex half-wave retarder has little or no effect on the components of these optical transfer matrices.  $M_{00}$  and  $M_{33}$  have the same amplitude and opposite phase. The images of  $M_{12}$ ,  $M_{21}$  and  $M_{22}$  can be obtained by rotating  $M_{11}$ .

## 5. COMPARISON OF OPTICAL TRANSFER MATRIX BETWEEN HALF-WAVE PLATE AND VORTEX HALF-WAVE RETARDER

We compared the Modulation Transfer Function(MTF) of the zero-order vortex half-wave retarder  $M_{00}$  with the modulation transfer function of the half-wave plate  $M_{00}$ , and the results are shown in Figure 3(I). In order to see the comparison results more clearly, we intercepted a section of the curve in the figure, as shown in Figure 3(II).

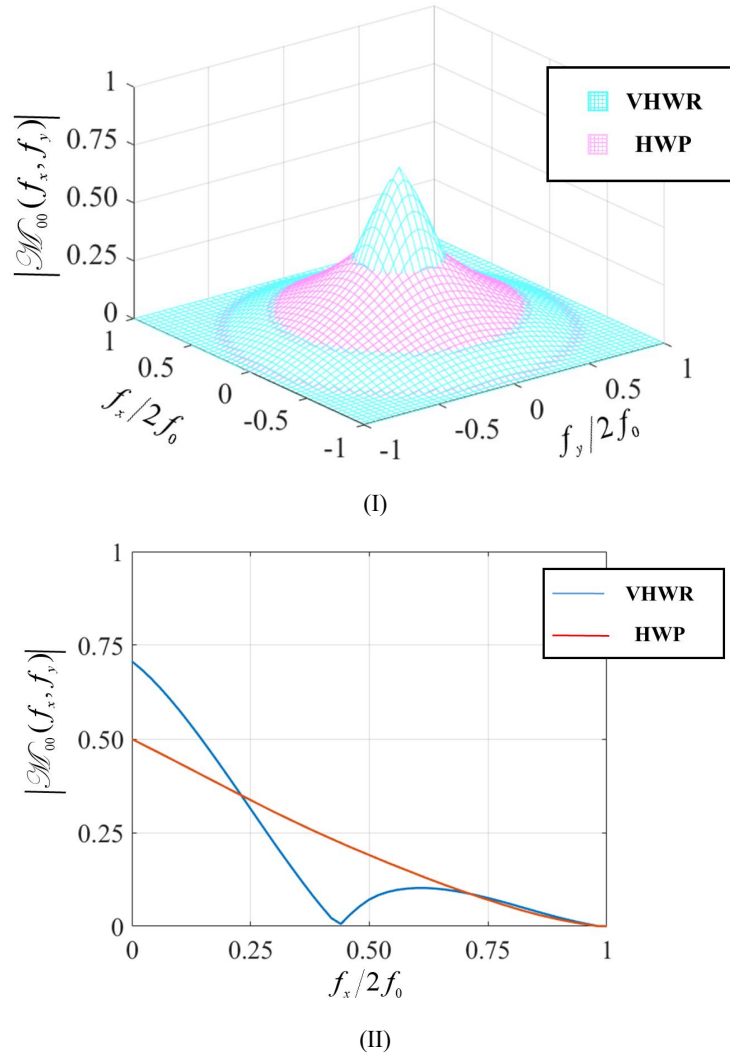


Figure 3. The  $M_{00}$  comparison image of MTF of HVWR and HWP, (I) 3D contrast image  $M_{00}$  of between HVWR and HWP.

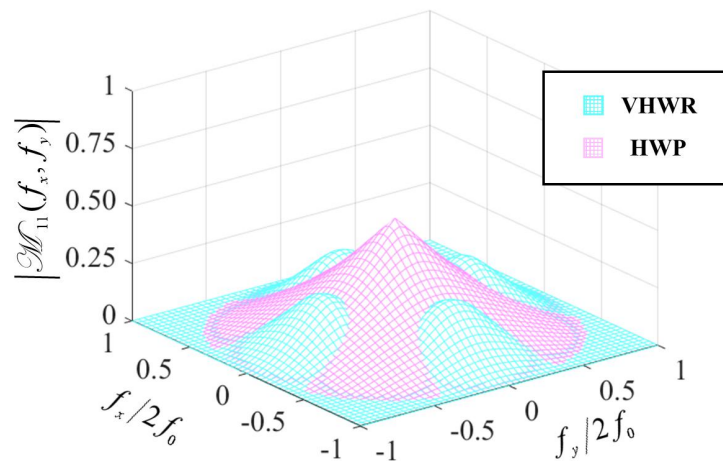
(II) 2D contrast curve  $M_{00}$  of between HVWR and HWP.

It can be seen from the above Figure 3 that in the range of low frequency and high frequency, the amplitude of the  $M_{00}$  component of the vortex half-wave plate is greater than that of the half-wave plate, indicating that in this case, the vortex half-wave plate can transmit more information. The greater the contrast, the better the image quality.

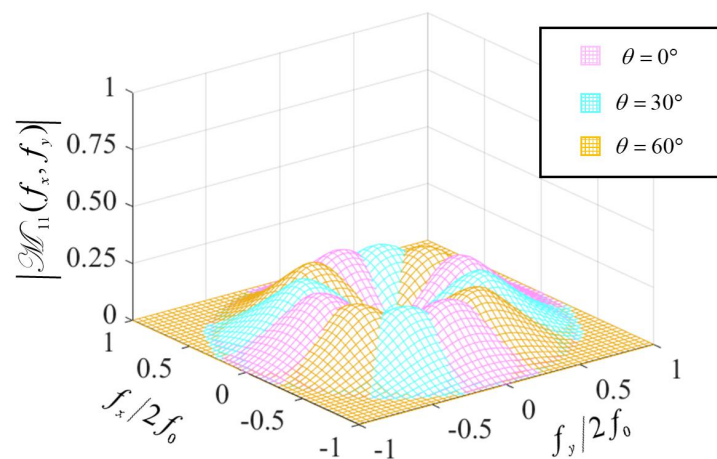
Meanwhile, we compared the Modulation Transfer Function (MTF) of the zero-order vortex half-wave retarder  $M_{11}$

with the modulation transfer function of the half-wave plate  $M_{11}$ , and the results are shown in Figure 4(I). The limitation of the  $M_{11}$  component of the vortex half-wave plate is that its maximum value is only obtained in a few places, and it is not a circularly symmetric structure. This problem can be solved by the rotation method. Figure. 4(II) is an image obtained by rotating the  $M_{11}$  component image of VHR by  $30^\circ$  and  $60^\circ$ . In order to see the comparison results more clearly, Taking the curve of the highest peak of the zero-order vortex half-wave plate  $M_{11}$  and the curve corresponding to the ordinary half-wave plate  $M_{11}$ , as shown in Figure 4(III).

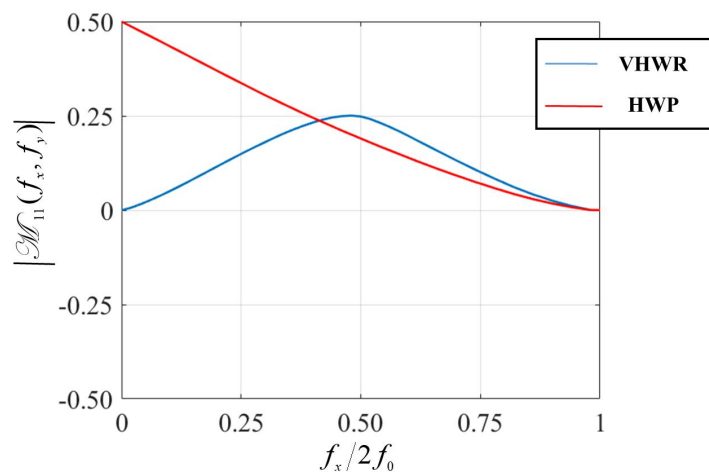
It can be seen from Figure 4 that in the high-frequency part, the amplitude of  $M_{11}$  of the vortex half-wave plate is larger than that of the ordinary half-wave plate, and the high-frequency part reflects the transmission of the optical system to the details of the object. This also proves that the use of vortex half-wave retarders in the polarization imaging system can indeed improve the resolution of the imaging system.



(I)



(II)



(III)

Figure 4. The  $M_{11}$  comparison image of MTF of HVWR and HWP; (I) 3D contrast image  $M_{11}$  of between HVWR and HWP. (II)The image obtained by rotating the  $M_{11}$  component image of VHR by  $30^\circ$  and  $60^\circ$ . (III) 2D contrast curve  $M_{11}$  of between HVWR and HWP.

## 6. CONCLUSION

In conclusion, we give the Jones matrix of the zero-order vortex half-wave retarder, and draw the image of the optical transfer matrix through the expression. Finally, the image of the optical transfer matrix of the zero-order vortex half-wave retarder is compared with the image of the optical transfer matrix of the half-wave plate. From the result, the value of the modulation transfer function of  $M_{00}$  and  $M_{11}$  in the zero-order vortex half-wave retarder in the high-frequency part is larger than the value of the modulation transfer function of the half-wave plates  $M_{00}$  and  $M_{11}$  in the high-frequency part. The high-frequency part of the optical transfer matrix reflects the system's detailed transfer of objects. Therefore, this paper proves that the use of vortex half-wave retarder can indeed improve the imaging resolution of the system.

## REFERENCES

- [1] R. Dorn, S. Quabis, and G. Leuchs, Phys. Rev. Lett. 91, 233901 (2003).
- [2] X. Grosjean, D. Courjon, and C. Bainier, Opt. Lett. 32, 976 (2007).
- [3] Scott C. McEldowney, David M. Shemo, Russell A. Chipman, Opt. Lett. 33, 88916 (2008).
- [4] T. Grosjean, F. I. Baida, and D. Courjon, "Conical optics: the solution of confine light," Appl. Opt.(2007).
- [5] Cheng Zheng, Guangyuan Zhao, Cuifang Kuang, Opt. Lett. 42, 303670 (2017).
- [6] J. W. Goodman, [Statistical Optics], Wiley-interscience, Colorado (2000).
- [7] Tianyu Zhao, Xing Zhou, Dan Dan, Acta Phys.66, 148704(2017).
- [8] Jing Bu, Lichao Zhang, Xiujie Dou, IRLA.46, 0634001(2017).
- [9] Hucheng He, Yiqun Ji, Jiankang Zhou, Optik.124 6857-6860 (2013).
- [10] Scott C. McEldowney, David M. Shemo, Russell A. Chipman, Opt. Expr. 16, 7295 (2008).